$$dP = \rho_o c_p du \qquad \text{(along h = const)} \tag{13}$$

$$dE = (P/\rho_o c_u)du \tag{14}$$

These equations can be applied incrementally to observed wave shapes to relate velocities to stress, density, and energy. Note that Eqs. (12) and (13) are equivalent to the ordinary jump conditions in differential form (in Lagrangian coordinates) except that different phase velocities must be used. Only for steady state waves, discontinuous shock fronts or isentropic simple waves will these velocities be equal in general.

The mass velocity can be eliminated from Eqs. (12) and (13) to give the stress-density relation:

$$dP/d\rho = (\rho_o^2/\rho^2)c_p c_u, \qquad \text{along h = const.} \tag{15}$$

Clearly, this reduces to the acoustic relation whenever the phase velocity $c = c_p = c_u$. Similarly, we can write:

$$dE = (PdP/\rho_o^2 c_u c_p) \qquad \text{along h = const.} \tag{16}$$

Implementation of this method in experiments requires, at a minimum, measurements of pressure-time profiles at more than one location in the specimen. From these the velocities c_p can be obtained and mass velocity profiles deduced from Eq. (13). Thence, c_u values can be obtained and the density and energy calculated. Simultaneous particle-velocity measurements would be valuable and might enhance the precision but are not essential. Particle velocity measurements alone, without pressure measurements, would require an iterative procedure to deduce P and c_p and, although less convenient, could be used.

The data obtained are values of one-dimensional stress, density, and internal energy for each part of the wave as it progresses through the material. From this information one can fit various models to obtain a general relation for stress as a function of strain, energy, strain-rate, history, etc. The stress components parallel to the wavefront might also be inferred from a variety of such measurements or from independent hydrostatic data.

The relation between the phase velocities can be derived from

$$c_p - c_u = (\partial h/\partial t)_p - (\partial h/\partial t)_u$$

$$= (\partial h/\partial t)_p - [(\partial h/\partial t)_p + (\partial h/\partial p)_t(\partial P/\partial t)_u]$$

$$= c_p \frac{(\partial P/\partial t)_u}{(\partial P/\partial t)_h}$$

or

$$1 - c_u/c_p = \frac{(\partial P/\partial t)_u}{(\partial P/\partial t)_h};$$

Similarly, (17)

$$1 - c_p/c_u = \frac{(\partial u/\partial t)_p}{(\partial u/\partial t)_h}$$

Consequently, the phase velocities will be equal whenever the stress is a function of velocity only, or when the wavefront represents a discontinuity in stress or mass velocity.

There are two cases when stress is a function of velocity only - steady flow and isentropic simple waves. This is shown in the following two sections. We can conclude immediately, however, that whenever $P = P(u)$ the lines of constant phase are straight lines in the h,t plane. That is, $c_u = c_p = c_p(P)$ from Eq. (13), and $(\partial c_p/\partial h)_p = 0$.

B. Steady State